Problem Set 8 - LV 141.246 QISS - 11.6.2012

1. Persistent-current qubit

The persistent-current qubit consists of a loop containing three Josephson junctions. Two junctions are equal and the third is by a factor α smaller. Therefor its Josephson energy is αE_J and its charging energy E_c/α.

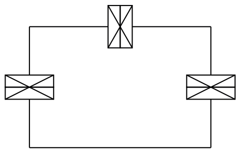

Figure 1: Persistent-current qubit

(a)

$$I_1 = I_0 \sin(\varphi_1) + C\dot{V}_1 = I_0 \sin(\varphi_1) + C\frac{\phi_0}{2\pi}\ddot{\varphi}_1$$

Similar equations hold for the other two junctions. Note the the phase difference of the third junction is $\varphi_2 - \varphi_1 + 2\pi\phi_{ext}/\phi_0$

$$I_1 = I_3 = -I_2$$

Find the Langrange function

$$\mathcal{L}(\varphi_1, \varphi_2, \dot{\varphi}_1, \dot{\varphi}_2)$$

and express it in terms of $E_J, E_c, \alpha, \phi_{ext}$

Show that the Lagrange equations

$$\frac{d}{dt}\left\{\frac{\partial \mathcal{L}}{\partial \dot{\varphi}_i}\right\} - \frac{\partial \mathcal{L}}{\partial \varphi_i} = 0$$

reproduce the Kirchhoff equations.

(b) Find the canonical conjugate variables q_i with

$$q_i = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_i}$$

and express the variables $\dot{\varphi}_i$ in terms of these new conjugate variables.

(c) Find the Hamilton function by doing a Legendre transformation

$$\mathcal{H}(\varphi_1, \varphi_2, q_1, q_2) = \dot{\varphi}_1 q_1 + \dot{\varphi}_2 q_2 - \mathcal{L}$$

Verify your result by making sure that the canonical equations reproduce the initial Kirchhoff equations.

$$\dot{\varphi}_i = \frac{\partial \mathcal{H}}{\partial q_i} \qquad \dot{q}_i = -\frac{\partial \mathcal{H}}{\partial \varphi_i}$$